

Fundamentals of GPR Data Interpretation

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GPR raw data has all the original information, and we have to extract desired information included in it. The amount of the information cannot be improved by any signal processing, but the value or the quality of the information for users can greatly be improved by proper signal processing. However, if we do not understand the meaning of each processing, the processing can produce serious artifact. These artifacts mislead the interpretation of GPR data. This text is prepared to explain fundamental physics and mathematics, which are used for GPR signal processing.

1. Fourier analysis

1.1. 1-D Fourier transformation

Fourier transformation is the most important bases of the signal processing. We define a function in time domain as $f(t)$ and its spectrum in frequency domain as $F(\omega)$. Both functions are related by the Fourier transformation as:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega \quad (1.1.1)$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \quad (1.1.2)$$

where ω (rad/s) is an angular frequency and related to frequency f (Hz) by $\omega = 2\pi f$. $f(t)$ is a real function and $F(\omega)$ is a complex function. The power spectrum of this signal is defined as:

$$P(\omega) = |F(\omega)|^2 \quad (1.1.3)$$

The raw signal acquired by GPR system is a time domain function and can be represented by $f(t)$.

1.2. Linear System

When parameters of a system are time invariable, the input $x(t)$ and the output $y(t)$ of the system is related by an impulse response of the system function $h(t)$ by:

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau = x(t) \otimes h(t) \quad (1.2.1)$$

The operator \otimes , which is defined in the time domain, is convolution. The spectrum of $x(t)$, $y(t)$ and $h(t)$ are given as $X(\omega)$, $Y(\omega)$ and $H(\omega)$, and they are related in the frequency domain by:

$$Y(\omega) = H(\omega) \cdot X(\omega) \quad (1.2.2)$$

where $H(\omega)$ is the system function, or the transfer function of a system..

1.3. 1-D Filtering in frequency domain

When we need to enhance or reject a specified spectrum component of a signal, we use a filter. Filtering in the frequency domain is described by using the notation of a linear system. When the spectrum of the raw signal is $X(\omega)$, the spectrum of the processed signal is $Y(\omega)$, and the filtering function is $H(\omega)$, the filtering operation is given by

$$Y(\omega) = H(\omega) \cdot X(\omega) \quad (1.3.1)$$

which is the same operation given in Eq.(1.2.2). This filtering is defined in the frequency domain, but the equivalent operation can be processed to the time domain signal by convolution defined in Eq.(1.2.1). This operation is directly carried out in digital filtering, but normally the time domain signal is Fourier transformed in to the spectrum, and after the operation of Eq.(1.3.1) the spectrum of the output signal is inverse Fourier transformed into the time domain signal again. This process is normally much faster than convolution, because we can use FFT in Fourier transformation.

(Ref: RADPRO: 7-7. Filter in Frequency Domain)

1.4. 1-D Filtering in spatial frequency domain

In the previous sections, we have discussed only the signal in the time-domain. However, all the discussion can be applied to any one-dimensional signal. For example, we think a height of a ground surface along a straight line. If we define the x-axis along the straight line, the height of the ground surface d can be represented as $d(x)$. Similarity to Eqs.(1.1.1)and (1.1.2), we can define the Fourier transformation of $d(x)$ as:

$$D(k_x) = \int_{-\infty}^{\infty} d(x)e^{-jk_x x} dx \quad (1.4.1)$$

$$d(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} D(k_x)e^{jk_x x} dk_x \quad (1.4.2)$$

where $D(k_x)$ is the spatial spectrum of $d(x)$ and k_x is the spatial wave number and

$$k_x = 2\pi/\lambda_x \quad (1.4.3)$$

where λ_x is the spatial wave length in the x-direction. The spatial spectrum $D(k_x)$ represents the intensity of the sinusoidal component having the wavelength of λ_x . Filtering in the spatial frequency domain is useful when we need to enhance or reject specified spatial frequency component.

(Ref: RADPRO: 7-8. Filter in Wave number Domain)

1.5. 2-D Fourier transformation

We can easily expand the One-dimensional Fourier transformation for 1-D signal into Two-dimensional Fourier transformation for 2-D signal. If we define a Two-dimensional signal as $f(t_1, t_2)$, 2-D Fourier transformation is defined as:

$$F(\Omega_1, \Omega_2) = \iint f(t_1, t_2) e^{-j(\Omega_1 t_1 + \Omega_2 t_2)} dt_1 dt_2 \quad (1.5.1)$$

$$f(t_1, t_2) = \frac{1}{2\pi j} \iint F(\Omega_1, \Omega_2) e^{j(\Omega_1 t_1 + \Omega_2 t_2)} d\Omega_1 d\Omega_2 \quad (1.5.2)$$

where t_1, t_2 can be any parameters such as time and spatial axis. Accordingly, Ω_1, Ω_2 are corresponding angular frequencies.

We can understand that the Kernel function $e^{j(\Omega_1 t_1 + \Omega_2 t_2)}$ in Eq.(1.5.2) is a fundamental function corresponding to the angular frequency (Ω_1, Ω_2) and it is weighted by Fourier spectrum $F(\Omega_1, \Omega_2)$ in the original 2-D signal.

$e^{j(\Omega_1 t_1 + \Omega_2 t_2)}$ is a complex function and we can expand it as:

$$e^{j(\Omega_1 t_1 + \Omega_2 t_2)} = \cos(\Omega_1 t_1 + \Omega_2 t_2) + j \sin(\Omega_1 t_1 + \Omega_2 t_2) \quad (1.5.3)$$

Fig. 1.5.1 show the real component of Eq.(1.5.3) $\cos(\Omega_1 t_1 + \Omega_2 t_2)$ for an angular frequencies (Ω_1, Ω_2) .

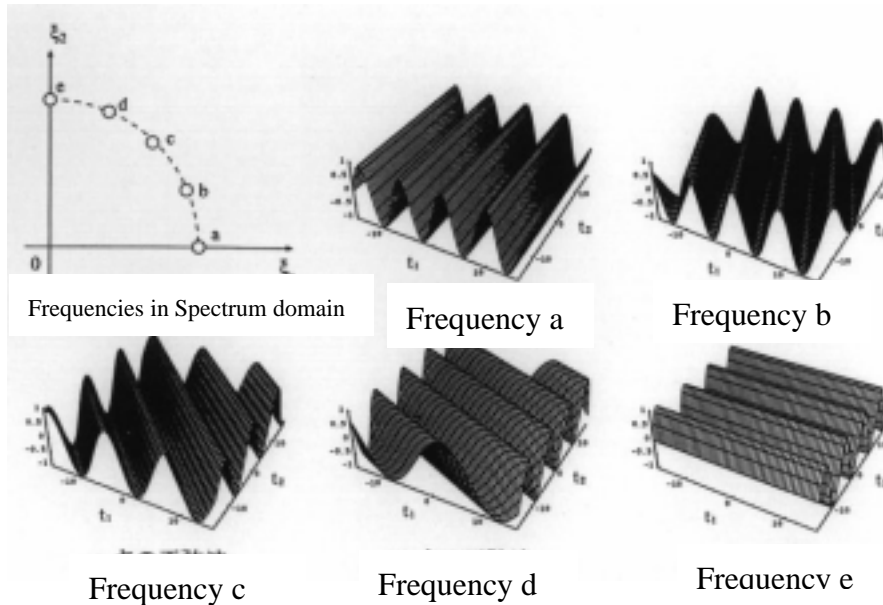


Fig. 1.5.1 Angular frequency (Ω_1, Ω_2) and corresponding

We can observe that this function is a sinusoidal function having the direction along the axis of

$$z = \Omega_1 t_1 + \Omega_2 t_2 \quad (1.5.4)$$

and the wavelength along the z -axis is defined by

$$\lambda = \frac{2\pi}{\sqrt{\Omega_1^2 + \Omega_2^2}} = \frac{2\pi}{\beta} \quad (1.5.5)$$

and the wave lengths along t_1 and t_2 axes are defined by:

$$\lambda_1 = \frac{2\pi}{\Omega_1}, \lambda_2 = \frac{2\pi}{\Omega_2} \quad (1.5.6)$$

and
$$\beta^2 = \Omega_1^2 + \Omega_2^2. \quad (1.5.7)$$

On the other hand, the direction of the wave propagation z is perpendicular to the constant phase plane in the t_1, t_2 domain, which is defined by:

$$z = \Omega_1 t_1 + \Omega_2 t_2 = \text{constant} = c, \quad t_2 = -\frac{\Omega_1 t_1}{\Omega_2} + \frac{c}{\Omega_2} \quad (1.5.8)$$

1.6. GPR radar profile as 2-D signal

When we acquire GPR signal along a survey line x , the radar profile is a 2-D signal and it can be represented by $f(t, x)$. This is the most common format of GPR raw data. Using Eqs.(1.5.1) and (1.5.2) we can define 2-D Fourier transformation of GPR data as:

$$F(\omega, k_x) = \iint f(t, x) e^{-j(\omega t + k_x x)} dt dx \quad (1.6.1)$$

$$f(t, x) = \frac{1}{2\pi} \iint F(\omega, k_x) e^{j(\omega t + k_x x)} d\omega dk_x \quad (1.6.2)$$

We can understand that $F(\omega, k_x)$ is a spectrum component having a spatial wave number k_x and angular frequency ω of the original 2-D signal $f(t, x)$.

1.7. 2-D Filtering in f-k domain

Originally in the seismic signal processing, but also in GPR signal analysis, $F(\omega, k_x)$ is normally referred as frequency-wave number or f - k spectrum.

According to Eq.(1.5.8)

$$x = -\frac{\omega t}{k_x} + \frac{c}{k_x} = -v_x t + x_0 \quad (1.7.1)$$

is the constant phase plane, where

$$v_x = \frac{\omega}{k_x} \quad (1.7.2)$$

is the apparent velocity of the wave along the x -axis.

From Eq.(1.7.2) we can understand that the wave

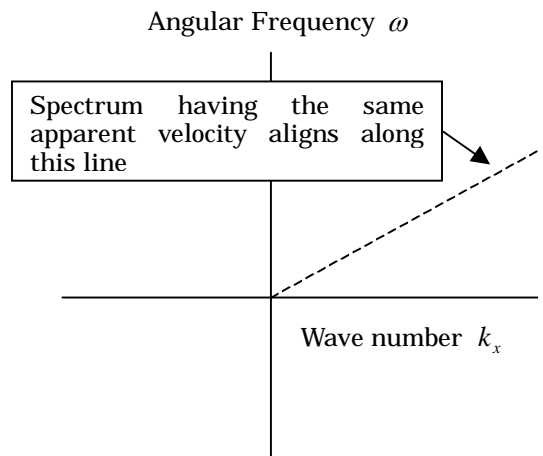


Fig.1.7.1 f - k domain

propagating to the same direction has its spectrum on the line which has the fixed angle in the $f-k$ domain. For instance, the directly coupled signal between the transmitter and the receiver has a constant arrival time independent on the antenna position x . This signal looks like propagating to the direction perpendicular to the x -axis. In this case, the apparent velocity v_x is infinitely large, and the spectrum of this signal is concentrated on the ω -axis, i.e., $v_k = \infty, k_x = 0$. In the same manner, a train of reflected signals from an inclined boundary sometimes appears in GPR profile. This signal spectrum is distributed along an inclined straight line in $f-k$ spectrum.

These signals can be rejected by suppressing the spectrum near the line having the same apparent velocity in $f-k$ spectrum by 2-D filtering. This filtering is simply carried out by multiplying filtering function to the $f-k$ spectrum as:

$$G(\omega, k_x) = F(\omega, k_x) \cdot H(\omega, k_x) \quad (1.7.3)$$

where $H(\omega, k_x)$ is the filtering function. Similar to the 1-D signal case, the filtering to the raw GPR signal $f(t, x)$ can be carried out after Fourier transformation into the $f-k$ spectrum and after Eq.(1.7.3) is applied, $G(\omega, k_x)$ is inverse Fourier transformed and we can obtain the processed GPR profile $g(t, x)$.

2. Amplitude Operation

2.1. Gain Control

When the GPR recorded signal is $f(t)$, it generally contains strong directly coupled component propagating from a transmitter to a receiver in the early time and weak reflection signals from the subsurface anomalies in the late time. Our interest is the weak reflection signal, so we need amplify the late time signal to enhance the weak reflection. Here we define an amplifying function $s(t)$. $s(t)$ is a time-domain function and it have to be selected properly depending on the signal. The processed signal $g(t)$ is given by:

$$g(t) = f(t) \cdot s(t) \quad (2.1.1)$$

It should be noted that the multiplication is carried out between the time domain functions. Therefore this is not a linear operation, unless $s(t) = a$, and the processed signal cannot be recover the original information. Power spectrum is also changed by this operation.

(Ref: RADPRO: 7-4. Gain, 7-5. Custom gain)

2.2. AGC

When $s(t)$ is chosen so that the envelop of the $g(t)$ always has a unity, this gain control is called AGC (Automatic gain Control). This is a easy process and since the signal has always the constant

amplitude, small reflection in the late time can clearly be observed. Therefore it is often used to display GPR profile. However, as I already have noticed, this process destroys the original information of the signal. So, it should be applied only for displaying the GPR profile. Applying any signal processing after AGC produces only artifact.

(Ref: RADPRO: 7-2. AGC (Automatic Gain Control))

3. Spatial Filtering

3.1. Delete mean trace

Raw GPR signal is normally composed from a strong directly coupled signal $d(t, x)$ between the transmitter and the receiver and the continuing reflection signals $r(t, x)$. When the ground condition does not change a lot along a survey line, $d(t, x)$ does not change, when the antenna separation is fixed. This condition is satisfied when we acquire the data by the profile (common offset) measurement. The raw signal measured at the antenna position x_i can be represented by:

$$f(t, x_i) = d(t, x_i) + r(t, x_i) \quad (3.1.1)$$

If we take the average of the N traces,

$$\bar{f}(t) = \frac{1}{N} \sum_{i=1}^N [d(t, x_i) + r(t, x_i)] \cong d(t, x) \quad (3.1.2)$$

this gives a good replica of the $d(t, x)$, because $r(t, x)$ is random in each x_i . Subtracting the averaged signal from the original signals approximately gives only $r(t, x)$. Therefore, the reflection signal can be enhanced.

$$f(t, x_i) - \bar{f}(t) \cong r(t, x_i) \quad (3.1.3)$$

However, we have to notice that this process contains a factor of low pass filtering in spatial frequency domain, and sometimes the processed signal looks de-focused.

(Ref: RADPRO: 7-13. Delete Mean Trace)

3.2. Moving Average

The moving average of $2N+1$ traces is defined as:

$$f(t, x_i) = \frac{1}{2N+1} \sum_{n=-N}^N a_n f(t, x_{i+n}) \quad (3.2.2)$$

This process is almost equivalent to the low pass filtering in spatial frequency domain. The filter

shape can be changed by the selection of the weighting function coefficient a_n .
 (Ref: RADPRO: 7-14. Moving Average Filtering)

4. Velocity Analysis

4.1. Velocity Fitting to a Point reflector

In profile measurement, if we have a point reflector at $(x = x_0, z = d)$ and the GPR is located at x on the ground surface $d = 0$, the travel time is given by:

$$\tau = \frac{2\sqrt{(x - x_0)^2 + d^2}}{v} \quad (4.1.1)$$

Eq.(4.1.1) contains three unknown functions, namely x_0, d, v . When we observe GPR profile, x_0 can be easily determined by the location of the apex of the hyperbolic curve. By changing d and v and fit the theoretical trace to the raw data, we can simultaneously estimate these two values. This technique is especially effective for point targets detection such as buried pipes and cables.

4.2. Velocity Fitting to a Linear Reflector

In CMP measurement, if we have a linear reflector such as a horizontal geological formation boundary at the depth of d , the travel time is given by:

$$\tau = \frac{2\sqrt{(\Delta x/2)^2 + d^2}}{v} \quad (4.2.1)$$

where Δx is the antenna separation. This theoretical trace is fitted to the acquired trace by changing the d and v , we can estimate these two values simultaneously.

4.3. Velocity Spectrum

Fig.4.3.1(a) shows a typical CMP gathers. When we assume a velocity, we can calculate the theoretical arrival in each trace and shift the arrival time so that all the traces arrives at the same time and all the traces aligned horizontally. This is the Normal Move out (NMO). If the assumed velocity is different from the actual velocity, the shifted traces cannot be aligned as Fig.4.3.1(b), but has different forms as Figs.4.3.1(c) and (d). If we take the accumulated energy along the horizontal axis in NMO gathers, it can be used as the measure of the velocity accuracy, because if the velocity is assumed correctly, the traces are horizontally aligned and gives the maximum energy.

By changing the velocity and takes the accumulated energy, we can obtain the velocity spectrum. Fig. 4.3.2 shows an example of the velocity spectrum. If we find the maximum value at each time, we can estimate the velocity at each depth. This gives the vertical velocity profile.

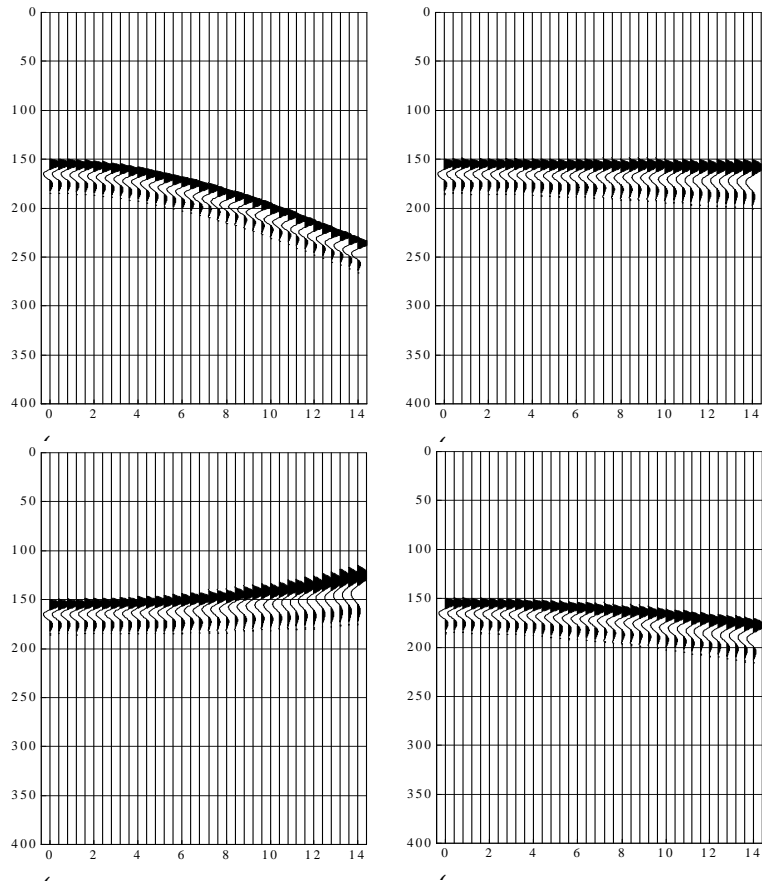


Fig.4.3.1 CMP data and NMO correction

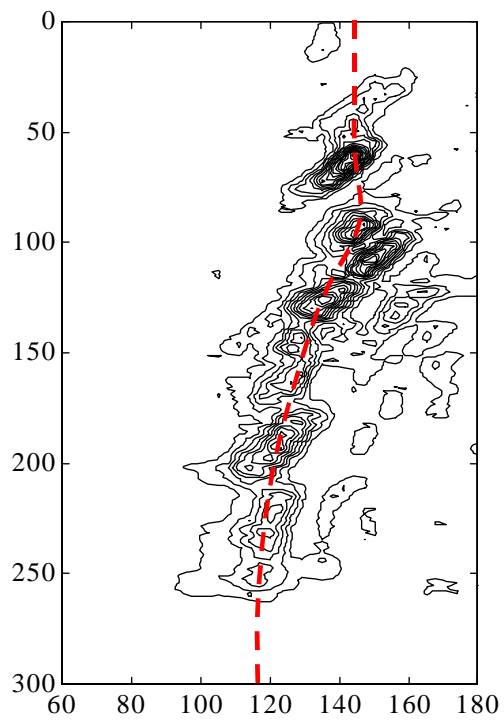


Fig. 4.3.2 Velocity Spectrum

5. NMO correction

5.1. NMO correction

The NMO (normal moveout) correction compensates the effect of the offset between Tx and Rx antennas. It converts the data acquired by the real bistatic antenna system into that by the virtual monostatic one. If we denote the reflector as d , the antenna offset as x , and the velocity of host medium as v , the two-way travel time t will be

$$t = \frac{2\sqrt{(x/2)^2 + d^2}}{v} \quad (5.1.1)$$

Therefore, the travel time is nonlinear to the reflector depth d . If we know or assume the velocity, we can easily compensate the effect of antenna offset x , in other words make x as zero, and then travel time is converted to linear equation of the reflector depth d . If we do not apply the NMO correction to bistatic antenna data, the depth axis must be expressed in nonlinear scale. Usually, we use the term “NMO correction”, when we correct the effect of Tx and Rx offset in multi-channel seismic data. Since NMO correction means correcting the effect of the offset distance between the source and receiver basically, I use the term “NMO correction” also to the single channel GPR data processing.

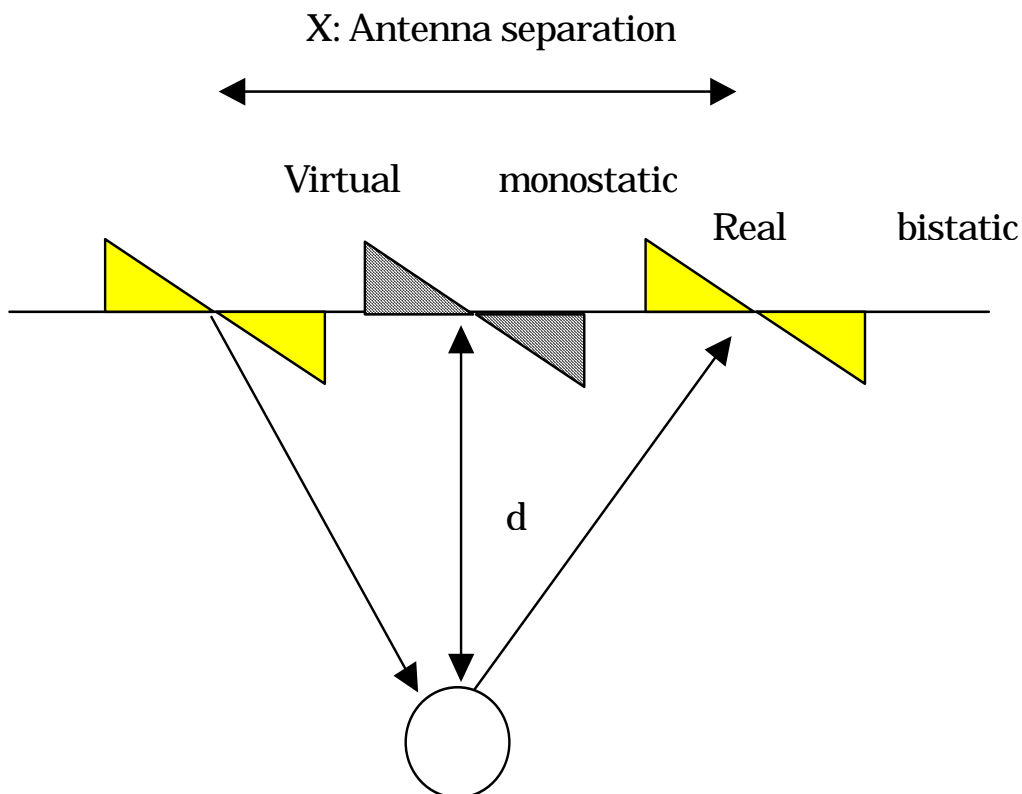


Fig.5.1 NMO correction

6. f - k Migration

6.1. Wave Equation in time-space domain

The scattering from radar targets are diffracted and the acquired raw GPR profile is not the same image as the actual vertical profile of subsurface structure. In order to estimate the true vertical subsurface image, migration process is used. Some methods for migration is known, but here we introduce f - k migration, which is effective in a homogeneous material.

Here we assume that the reflection wave is a wave radiated from a source located at the reflection point at $t=0$, and the field is expressed as:

$$u(x, y, t) \quad (6.1.1)$$

We assume the wave velocity as:

$$v = \frac{c}{2\sqrt{\epsilon_r}} \quad (6.1.2)$$

The scalar wave equation can be written as:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2} = 0 \quad (6.1.3)$$

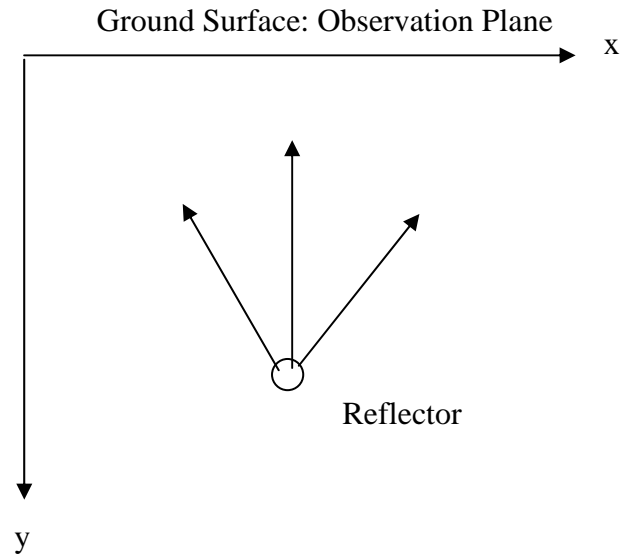


Fig. 6.1.1 Geometry for Migration

6.2. Wave Equation in Spectrum Domain

Here we assume the time dependence of $e^{j\omega t}$ and x-direction dependence as $e^{jk_x x}$. Using the 2-D Fourier spectrum $U(k_x, y, \omega)$ of $u(x, y, t)$ Eq.(6.1.3) can be re-written as:

$$\frac{\partial^2 U}{\partial y^2} - k_x^2 U + \frac{\omega^2}{v^2} U = 0 \quad (6.2.1)$$

and
$$\frac{\partial^2 U(k_x, y, \omega)}{\partial y^2} = -\left(\frac{\omega^2}{v^2} - k_x^2\right)U(k_x, y, \omega) \quad (6.2.2)$$

The solution to this differential equation is given as:

$$U = U_1 e^{-jk_y y} + U_2 e^{+jk_y y} \quad (6.2.3)$$

where
$$k_y = \sqrt{\frac{\omega^2}{v^2} - k_x^2} \quad (6.2.4)$$

6.3. Migration Process

The GPR data is acquired on the ground surface, i.e., along $y = 0$, and

$$u(x,0,t) \quad (6.3.1)$$

is observed value. Taking the 2-D Fourier transformation to Eq.(6.3.1) we have:

$$U(k_x,0,\omega) \quad (6.3.2)$$

This spectrum is to be used as initial condition of the wave field. By using Eq.(6.2.3) and apply Eq.(6.3.1) we have

$$U(k_x,0,\omega) = U(k_x,y,\omega)U_1 e^{-jk_y y} \quad (6.3.3)$$

which correspond to the up-going wave. Then we obtain the spectrum of the wave in arbitrary position as:

$$U(k_x,y,\omega) = U(k_x,0,\omega) e^{+jk_y y} \quad (6.3.4)$$

By taking the inverse Fourier transformation of Eq.(6.3.4), we can obtain the wave filed in the time-space domain as:

$$u(x,y,t) = \iint U(k_x,0,\omega) e^{+jk_y y} e^{+jk_x x} e^{j\omega t} dk_x d\omega \quad (6.3.5)$$

We calculate Eq.(6.3.5) for $t=0$ in order to visualize the reflection points.

$$u(x,y,0) = \iint U(k_x,0,\omega) e^{+jk_y y} e^{+jk_x x} dk_x d\omega \quad (6.3.6)$$

where, by using (6.2.4)

$$\omega^2 = (k_x^2 + k_y^2)v^2 \quad (6.3.7)$$

$$\text{and } d\omega = \frac{k_y}{\omega} v^2 dk_y = \frac{k_y}{\sqrt{k_x^2 + k_y^2}} dk_y \quad (6.3.8)$$

Substituting Eq.(6.3.8) into Eq.(6.3.6), we obtain:

$$u(x,y,0) = \iint U(k_x,0,v\sqrt{k_x^2 + k_y^2}) e^{+jk_y y} e^{+jk_x x} \frac{k_y}{\sqrt{k_x^2 + k_y^2}} dk_y dk_x \quad (6.3.9)$$

This gives the reflection point image, i.e., migrated image.

6.4. Summary

The following shows the summary of the f - k migration process:

- (1) Wave field observed on the ground surface: $u(x,0,t)$
- (2) 2-D Fourier transformation as for x and t : $U(k_x,0,\omega)$
- (3) Fourier spectrum at the arbitrary position: $U(k_x, y, \omega) = U(k_x, 0, \omega)e^{+jk_y y}$
- (4) Obtain $u(x, y, 0)$ by Eq.(6.3.6)

It should be noted that before the migration processing, the GPR trace must be adjusted zero time, and NMO correction should be carried out.

The velocity estimation is also very important to obtain the correct migrated image. Fig.6.4.1 shows the synthesized GPR profile, which is a reflection from a point target. Due to wide angle radiation of GPR, the reflection wave shows a hyperbolic curve and is different from the original shape of the point target. Fig. 6.4.2(a) shows the f - k migrated image with a correct velocity. The image is converged to one point and we can clearly see the effect of migration. Fig.6.4.2(b) show she migrated image with 33% slower velocity and Fig.6.4.2(c) shows that of 33% faster velocity. We can observe the artifact crated in the migrated image, when the assumption of the velocity is incorrect.

(Ref: RADPRO: 7-16. Migration by Constant velocity)

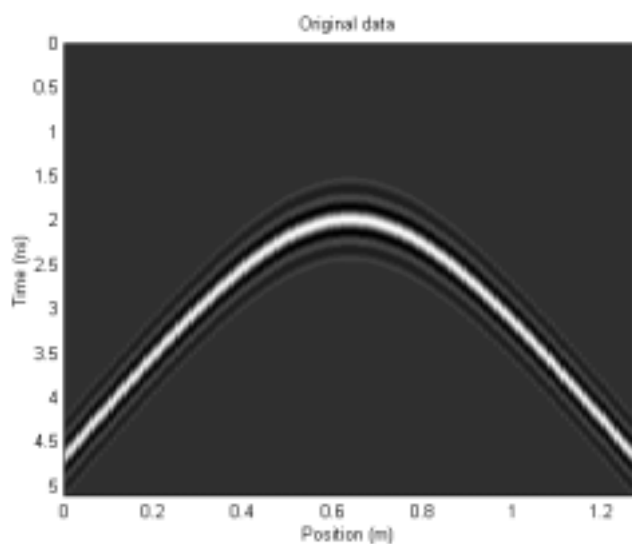


Fig. 6.4.1 Synthesized GPR profile from a point reflector

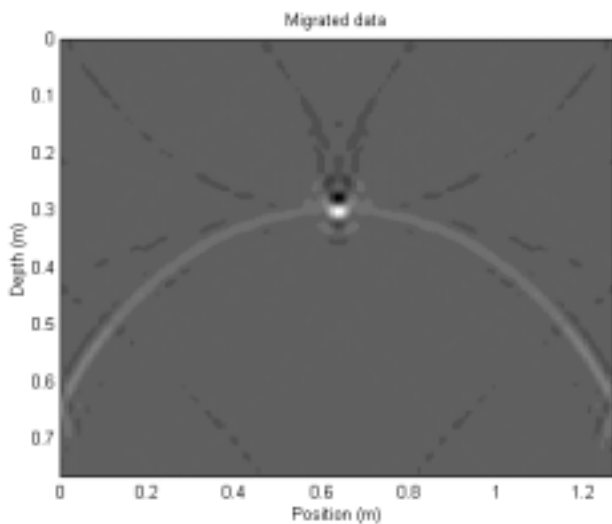


Fig. 6.4.2(a) Migrated image with a correct velocity

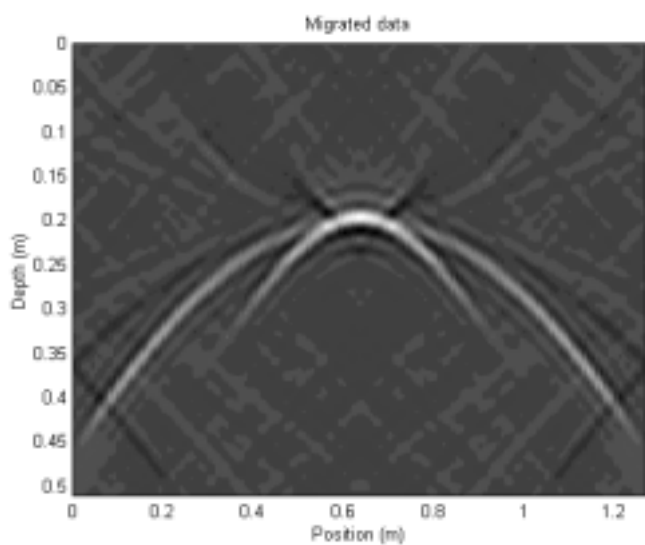


Fig.6.4.2(b) Migrated image with a 30% slower velocity

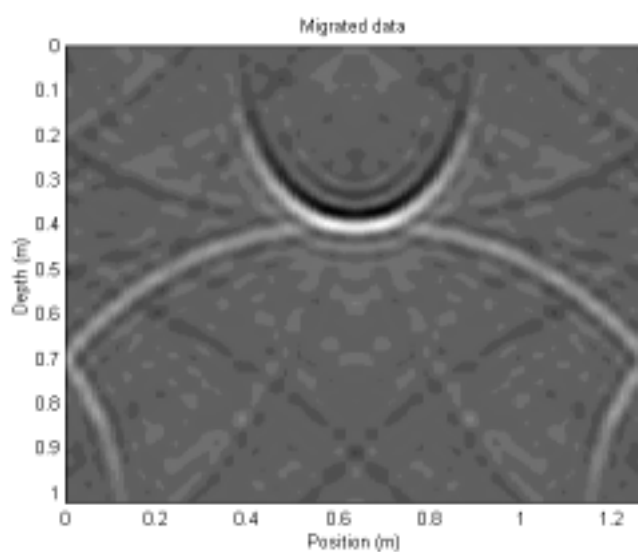


Fig.6.4.2(c) Migrated image with a 30% faster velocity

7. Conclusion

This text covers the major part of signal processing used for GPR, but the signal processing has much wider aspects. GPR signal processing is based on time-sequential signal processing and image processing, that corresponds to 1-D Fourier transformation and 2-D Fourier transformation.

I believe GPR signal processing is very typical example of signal processing, so it is not only for GPR users, but also it is very valuable examples for other fundamental engineering fields such as digital signal processing (DSP) and information science.

References:

Jung-Ho Kim, RADPRO/GPR V.3.0 User's Guide